Harmonic Contamination Of IMD

Draft

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6 II 94, rev. 13 VII 07 and 5/20/2011

About twelve years ago it occurred to me that harmonic contamination might invalidate 2nd and 3rd order intermodulation distortion measurements. There have been tacit suggestions of that possibility prior to 1994 and explicit suggestions subsequently. At that time, 12 years ago, I decided to try to derive formulas from which the harmonic cobtamination of 2nd and 3rd order intermodulation distortion could be predicted. As is evident, progress has been slow. Part of the reason is that I thought I had correctly analyzed the problem long ago. However, that now appears not to have been the case. Recently I began to suspect that that some of the formulas which I had derived previously were not correct when I was unable to verify the formulas experimentally. I used two HP 8640B signal generators and a hybrid combiner similar to one recommended by Hayward in his ARRL book ***Solid State Design*** as my two tone IMD measurement system, and a third signal generator and combiner to inject and vary harmonic content. The results of my experiments indicated my previous formulas were wrong. Examination of the derivations of those formulas showed that the derivations were wrong. So here we go again.

The usual mathematical model of intermodulation distortion takes the first few terms of the Maclaurin series expansion of the transfer function,

$$V_{out} = V_0 + k_1 V_{in} + k_2 (V_{in})^2 + k_3 (V_{in})^3,$$

assumes an input signal to the DUT of the form

$$V_{in} = E_1 \cos(\omega_1 t) + E_2 \cos(\omega_2 t),$$

expands the input signal applied to the transfer function, and after application of trig identities and rearrangement of terms, develops the following output function:

$$V_{out} = V_0 + 1/2\, k_2[E_1^2 + E_2^2]$$

$$+ [k_1E_1 + 3/4\, k_3E_1^3 + 3/2\, k_3E_1E_2^2] \cos(\omega_1 t)$$

$$+ [k_1E_2 + 3/4\, k_3E_2^3 + 3/2\, k_3E_1^2E_2] \cos(\omega_2 t)$$

$$+ 1/2\, k_2E_1^2 \cos(2\omega_1 t)$$

$$+ 1/2\, k_2E_2^2 \cos(2\omega_2 t)$$

$$+ k_2E_1E_2 \cos((\omega_1 + \omega_2)t)$$

$$+ k_2E_1E_2 \cos((\omega_1 - \omega_2)t)$$

$$+ 1/4\, k_3E_1^3 \cos(3\omega_1 t)$$

$$+ 1/4\ k_3 E_2^3 \ COS(3\omega_2 t)$$

$$+ 3/4\ k_3 E_1^2 E_2 \ COS((2\omega_1 + \omega_2)t)$$

$$+ 3/4\ k_3 E_1^2 E_2 \ COS((2\omega_1 - \omega_2)t)$$

$$+ 3/4\ k_3 E_1 E_2^2 \ COS((2\omega_2 + \omega_1)t)$$

$$+ 3/4\ k_3 E_1 E_2^2 \ COS((2\omega_2 - \omega_1)t).$$

A variant of the above formula was given in "Don't guess the spurious level of an amplifier. The intercept method gives the exact values with the aid of a simple nomograph," by F. McVay, ***Electronic Design*** 3, February 1, 1967, 70 – 73.

However, in practice the input to a DUT often does not have the above assumed form; it usually contains harmonics. The harmonics may originate in the signal generators of the two tone measurement system, or in some other component(s) of the systems, or in a combination of multiple sources preceding the DUT.

In general the input to a DUT from a two tone IMD measurement system may be assumed to have the form

$$V_{in} = E_1 \ COS(\omega_1 t) + F_1 \ COS(2\omega_1 t) + G_1 \ COS(3\omega_1 t) + \ldots$$

$$+ E_2 \ COS(\omega_2 t) + F_2 \ COS(2\omega_2 t) + G_2 \ COS(3\omega_2 t) + \ldots$$

provided the IMD output products of the measurement system which propagate through the DUT are negligible compared to the IMD produced in the DUT. To simplify the following development, we take only the fundamentals and 2nd harmonics, not the 3rd and higher harmonics.

$$V_{out} = [V_0 + 1/2\ k_2\{E_1^2 + E_2^2 + F_1^2 + F_2^2\} + 3/4\ k_3\ \{E_1^2 F_1 + E_2^2 F_2\}]$$

$$+ [k_1 E_1 + k_2 E_1 F_1 + 3/4\ k_3 E_1^3 + 3/2\ k_3 E_1 E_2^2 + k_3 E_1\{3/2\ F_1^2 + 1/2\ F_2^2\}] \ COS(\omega_1 t)$$

$$+ [k_1 E_2 + k_2 E_2 F_2 + 3/4\ k_3 E_2^3 + 3/2\ k_3 E_1^2 E_2 + k_3 E_2\{1/2\ F_1^2 + 3/2\ F_2^2\}] \ COS(\omega_2 t)$$

$$+ [1/2\ k_2 E_1^2 + k_3\{(E_1^2 + E_2^2)F_1 + 1/2\ F_1 F_2^2 + 3/4\ F_1^3\}] \ COS(2\omega_1 t)$$

$$+ [1/2\ k_2 E_2^2 + k_3\{(E_1^2 + E_2^2)F_2 + 1/2\ F_1^2 F_2 + 3/4\ F_2^3\}] \ COS(2\omega_2 t)$$

$$+ [k_2 E_1 E_2 + k_3 E_1 E_2\{F_1 + F_2\}] \ COS((\omega_1 + \omega_2)t)$$

$$+ [k_2 E_1 E_2 + k_3 E_1 E_2\{F_1 + F_2\}] \ COS((\omega_1 - \omega_2)t)$$

$$+ [k_2 E_1 F_1 + 1/4\ k_3\{E_1^3 + 3\ E_1 F_1^2\}] \ COS(3\omega_1 t)$$

$$+ [k_2 E_2 F_2 + 1/4\ k_3\{E_2^3 + 3\ E_2 F_2^2\}] \ COS(3\omega_2 t)$$

$$+ [3/4\ k_3 E_1 E_2^2 + 3/2\ k_3 E_2 F_1 F_2\ ] \ COS((2\omega_1 + \omega_2)t)$$

$+ [3/4\ k_3E_1E_2^2 + 3/2\ k_3E_2F_1F_2]\ \mathrm{COS}((2\omega_1 - \omega_2)t)$

$+ [3/4\ k_3E_1E_2^2 + 3/2\ k_3E_1F_1F_2]\ \mathrm{COS}((2\omega_2 + \omega_1)t)$

$+ [3/4\ k_3E_1E_2^2 + 3/2\ k_3E_1F_1F_2]\ \mathrm{COS}((2\omega_2 - \omega_1)t)$

+ higher order terms.

IIP2 Contamination?

When the tones have equal powers, for the $k_3E_1E_2F_1$ or $k_3E_1E_2F_2$ coefficients to be more than 10 dB less than the $k_2E_1E_2$ coefficient of the $\mathrm{COS}((\omega_1 +/- \omega_2)t)$ terms, the condition

$-IIP2 + G + 2x > -2IIP3 + G + 2x + z + 12.5$

must be satisfied, where x and z are the tones and 2nd harmonics powers respectively. The condition simplifies to

$2IIP3 – IIP2 > z + 12.5$.

For exammple, if IIP3 = +30 dBm, IIP2 = 80 dBm, and z = -50 dBm, then the condition is satisfied, and 2nd harmonic contamination of the 2nd order input intercept is negligible (down 10 dB or more). In fact, the amount of contamination C can be calculated by

$C = 12.5 + z + IIP2 – 2IIP3$, which in this case is $C = -22.5$ dB.

In general we would not expect any observable 2nd harmonic contamination of IIP2 for typical small signal amplifiers and typical IMD measurement systems.

IIP3 Contamination?

When the tones have equal powers, for the $3/2\ k_3E_2F_1F_2$ coefficient to be more than 10 dB less than the $3/4\ k_3E_1E_2^2$ coefficient of the $\mathrm{COS}((2\omega_1 +/- \omega_2)t)$ terms, the condition

$x > z + 8$

must be satisfied. It is difficult to imagine a case where the harmonic content of the signal generators of an IMD measurement system would not satisy this condition.

No Contamination?

For a long time I thought that under any reasonable conditions the harmonic content of the signal generators of an IMD measurement system cannot contaminate the 2nd or 3rd order intercept measurements of an IMD measurement system. However, the model developed here does is not bidirectional, and so the issue is not yet analyzed correctly.